

Comments on “A simple model for simulating tornado damage in forests”

(A. P. Holland, A. J. Riordan, and E. C. Franklin, 2006: *JAMC*, **45**(12), 1597-1611)

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1 **1 Introduction**

2 Holland et al. (2006) present a very interesting study on development and evaluation of a
3 simple analytical model of tornado vortex flow and its impact on specified forest
4 configurations. The authors also make reference to earlier work by Johannes Letzmann on
5 near-surface tornado wind fields, dating back to 1923 and reviewed e. g. by Peterson (1992a).

6 The authors are correct to say that Letzmann did not include information on the
7 physics of tree response (which was unavailable at his time), even though he considered the
8 question if and how twisted tree snapping occurred or how the observed tree damage should
9 be interpreted. However, some other statements by Holland et al. (2006) about Letzmann's
10 work can be misleading. Certainly, the review by Peterson (1992a) alone is not sufficient to
11 fully assess the analytical model developed by Letzmann (1923) in his Ph.D. thesis and later
12 summarized in a journal article (Letzmann, 1925).

13 It appears as if Holland et al. (2006), based on the limited information they had
14 available on Letzmann's model, reinvented parts of it. Thus it comes as no surprise that some
15 of Holland et al.'s results "are somewhat analogous to the hand-drawn diagrams of
16 Letzmann" (p. 1598) – the underlying model is the same. The fuzzy wording by Holland et al.
17 (2006) may have been influenced by their references: Letzmann (1925) was cited by Hall and
18 Brewer (1959), yet they only referred to "somewhat similar" work by Letzmann, and Peterson
19 (1992a) mentions Letzmann's "hand calculations".

20 When Holland et al. (2006) refer to Letzmann's work as "experimenting" with various
21 model parameters, and emphasize several times his "hand-drawn" diagrams and "hand
22 calculations", the reader may get the false impression that Letzmann had received his results
23 merely by chance, instead of by the rigorous analytical calculations he performed in his Ph.D.
24 thesis and which also extend the wind field description by Holland et al. (2006). Furthermore,
25 hand calculations and hand-drawn diagrams were state-of-the-art in the 1920s and 30s, just as
26 publishing scientific work in German language was. Nevertheless, the authors must be highly
27 credited for their tying in with Letzmann's research and augmenting it by the modelling of
28 tree response.

29 The purpose of our comment is to draw attention to the full set of references to
30 Letzmann's work relevant here, and thereby to facilitate ongoing and future research on
31 tornado damage in forests and near-surface tornado wind fields. In Sec. 2, we sketch the
32 historical context under which Letzmann pursued his studies, briefly review his analytical
33 tornado wind field model, and call attention to his guidelines for tornado research and damage

1 surveys, which were approved by the International Meteorological Organisation in 1937.
2 Sec. 3 presents our conclusions.

3

4 **2 Letzmann's tornado research related to forest damage**

5 Forest damage has traditionally been taken into account when tornadoes or other severe wind
6 events were investigated in Europe, see e. g. Martins (1850), Reye (1872), Wegener (1917),
7 or very recently, the International Conference on Wind Effects on Trees in 2003 (see
8 www.ifh.uni-karlsruhe.de/science/aerodyn/windconf.htm) and Hubrig (2004). Thus, it was
9 quite natural that parts of Letzmann's work on tornadoes were devoted to this field.

10

11 **2.1 Inspiration by Alfred Wegener**

12 Johannes Letzmann's tornado research was significantly triggered and enhanced by the
13 inspiration he received from Alfred Wegener, nowadays mostly remembered for his work on
14 continental drift. However, Wegener was a dedicated and thoughtful scientist whose research
15 interests covered an immensely broad range in geophysics and meteorology, including
16 thunderstorms and tornadoes.

17 In his service during World War I, Wegener was injured. After recuperating, he started
18 to pursue a comprehensive monograph on tornadoes and waterspouts in Europe (Wegener,
19 1917), a classic of tornado research literature. Only recently, Dotzek (2003) was able to
20 update Wegener's estimate of tornado occurrence in Europe.

21 After Wegener's recovery, he was assigned as a weather advisor on the Eastern Front.
22 With the collapse of the Russian Empire, the Prussian government seized the opportunity to
23 re-establish a presence in the Baltic States, particularly at universities. So Wegener was
24 dispatched to the University at Dorpat (Tartu), Estonia in 1918. Here, Johannes Peter
25 Letzmann (1885-1971) was especially interested in storms and had synthesized a climatology
26 of thunder observations across the area (cf. Peterson, 1995). He soon came under the
27 mentorship of Wegener (Lüdecke et al., 2000). With the end of the war, Wegener however
28 shortly returned to Germany while Letzmann continued his studies of severe storms and
29 tornadoes.

30 From 1919 to 1924 Wegener headed the Meteorological Department of Deutsche
31 Seewarte in Hamburg. Frequent correspondence reveals that Wegener was extremely insistent
32 that Letzmann join him in Hamburg with a suitable research position. However he obtained a
33 teaching position at Dorpat University and remained there until the dawn of World War II.

1 His Ph.D. thesis (Letzmann, 1923), summarized by Letzmann (1925), contained
2 groundbreaking analytical work on near-surface tornado windfields and damage. Both
3 scientists maintained close collaboration on tornadoes and friendship until Wegener's death in
4 Greenland in 1930.

5 During the 1930s Letzmann had become a major tornado researcher, including theory
6 of vortex dynamics, details of earlier tornadoes, damage swath investigations, case histories,
7 photographic sequence analysis, and laboratory simulations. Along with Harald Koschmieder
8 and on behalf of the International Meteorological Organisation, he prepared guidelines to the
9 study of tornadoes which were officially resolved in 1937 but appeared in print only two
10 years afterwards (Koschmieder and Letzmann, 1939; Letzmann, 1939).

11 In 1940, Letzmann came to the University of Graz on the invitation of Kurt Wegener.
12 Here, he could establish a research facility for atmospheric vortices (*Forschungsstelle für*
13 *atmosphärische Wirbel*) and received the title of an adjunct professor. However, in late 1945
14 he lost his position at the university, and his professorship was withdrawn. With an
15 increasingly difficult private situation over his long-lasting struggle to re-establish his
16 lectureship, he moved to the German island of Langeoog in 1962, where he stayed in a facility
17 for elderly Baltic Germans. He continued studies on tornadoes until his death in 1971, but at
18 this time his work was nearly forgotten both in Germany and the USA.

19 Alfred Wegener was able to provide fruitful inspiration to the younger Letzmann by
20 his own visionary work on tornadoes. This led to a remarkable list of papers (cf. Peterson,
21 1992a) which gain their full value only today with availability of mobile Doppler radars (e. g.
22 Lee and Wurman, 2005), computer models (e. g. Lewellen et al., 1997), and detailed damage
23 assessments (e. g. Wurman and Alexander, 2005; Wurman et al., 2007) as envisioned by
24 Letzmann decades ago.

25

26 **2.2 Analytical near-surface tornado wind field model**

27 Letzmann had presented the full analytical derivation of his near-surface tornado wind field
28 model only in his Ph.D. thesis¹ (Letzmann, 1923). The summary which appeared two years
29 later in the *Meteorologische Zeitschrift* was detailed but less technical (Letzmann, 1925).
30 Letzmann started from the assumption that the velocity field in a tornado vortex could be
31 described by a Rankine vortex with a solid body rotation in the core up to the radius of

¹ As the Ph.D. thesis is not easy to obtain today, it has been made available online in digitised form on the ESSL website, and paper copies of the specimen from Letzmann's legacy are available from the first author upon request.

1 maximum winds, followed by a hyperbolic decay of both the tangential and radial wind
2 speeds for larger radii, called the “mantle” by Letzmann. The ratio between the two vortex
3 wind components determined the angle of deflection (*Ablenkungswinkel*) α . A further key
4 parameter describing the flow field was G , the ratio between rotational and translational
5 motion of the tornado, and finally, ψ denoted the angle between the local instantaneous wind
6 speed in a given point versus the direction of translation of the tornado.

7 This nomenclature was introduced by Peterson (1992a) in his review of Letzmann’s
8 work, and also Holland et al. (2006) apply it in their paper. Clearly, their wind field model
9 description is a reiteration of the main parts of Letzmann’s analytical model. While other
10 authors like Wurman and Alexander (2005) also assume Rankine-type vortices, only Holland
11 et al. (2006) directly follow the analytical formalism as set out by Letzmann (1923).

12 Holland et al. (2006) used velocity ratios G_{max} varying between 2.1 and 20, while
13 Letzmann (1923, 1925) started his range of parameters G already below the critical value G_{max}
14 = 1, and found that for $G_{max} < 1$, the flow field loses some of its vortex characteristics and
15 corresponds more to a wave pattern. Interestingly, Letzmann focused on the case $G_{max} \approx 6$,
16 which he assumed representative of tornadoes in the USA. Furthermore, we note that Holland
17 et al. (2006) only treat the case of $\alpha = \text{const.}$, while Letzmann allowed for values of α variable
18 with radius.

19 In creating the resulting streamline diagrams, Letzmann could rely on earlier work by
20 Sandström (1909), which he developed further to his “method of individual circles” (*Methode*
21 *der Individualkreise*). This technique allowed him to identify singular lines (*singuläre Linien*)
22 of convergence and divergence lines within the vortex, as well as the locations of calms. In
23 those vortices which contained a closed singular line, a “genuine core” (*echter Kern*) was
24 present if the singular line showed convergence on both sides (outflow in the centre of the
25 vortex), while a “false core” (*unechter Kern*) was present if the singular line showed a
26 convergence-divergence couplet (inflow at the vortex’ centre). In modern terminology, this
27 corresponds to the distinction between a two-cell and a one-cell tornado, respectively.

28 For $G_{max} \geq 1$, two other types of singular lines become discernible in the vortex: First,
29 a “separation line” (*Grenzlinie*) dividing two regions of the vortex in which the streamlines
30 enter the vortex from the rear side and either leave the vortex at the front side or converge into
31 the separation line. Second, a “blocking line” (*Sperrlinie*) which surrounds an area of the
32 vortex in which streamlines entering from the rear flank cannot reach the vortex front side,
33 but converge either towards the central calm or to the separation line. By identifying these

1 different lines and their locations in a reconstructed streamline diagram, Letzmann (1923,
2 1925) was able to study a wide range of specified vortex setups. It should further be noted that
3 his analysis was derived in principle for any kind of vortex, and he consequently treated
4 tropical and extratropical cyclones as well to underpin the general applicability of his
5 analytical approach.

6 Before turning towards Letzmann’s application of his method to forest damage
7 patterns, we finally address the issue of the “hand calculations” and “hand-drawn diagrams”
8 mentioned by Peterson (1992a) and Holland et al. (2006) in order to shed light on the
9 soundness of this method of streamline reconstruction. The technique was developed by
10 Sandström (1909), and the Letzmann legacy contains a later-published whole textbook on
11 graphical streamline reconstruction.

12 Thus, Letzmann (1925) refers to the “Sandström technique” which was likely
13 motivated by its relevance to produce streamline maps in synoptic meteorology: After
14 computing the isogone fields for a given flow field, the streamlines obeying the equation
15

$$\frac{dy}{dx} = f(x, y) \quad (1)$$

17 could be obtained graphically, or for a larger number of fields or a parameter study also
18 mechanically. Sandström (1909) describes a mechanical device (cf. Fig. 1a) which was
19 developed by his student V. Söderberg and which was able to graphically solve about 100
20 differential equations like Eq. (1) per day. Sandström (1909) presents a large number of
21 worked-out examples of idealised and synoptic isogone and streamline fields, of which we
22 show one quite complex specimen in Fig. 1b.
23

24

25 **2.3 Guidelines for tornado research and forest damage surveys**

26 Letzmann’s guidelines for the study of tornadoes were resolved by the IMO in September
27 1937 (Salzburg, 14 September 1937, Resolution IV) following earlier recommendations by
28 the Climatological Commission of the IMO to the member states to pay more attention to
29 tornadoes (Danzig, 1935. Publ. Nr. 25, p. 21). The authors are presently unaware if IMO
30 resolutions from that time still bear validity in the context of present-day WMO regulations.

31 The IMO guidelines from 1937 appeared in print two years afterwards² (Koschmieder
32 and Letzmann, 1939; Letzmann, 1939) and were only slightly revised later on by Letzmann

² Both Koschmieder and Letzmann (1939) and Letzmann (1939) are available online on the ESSL website.

1 (1944). After falling into oblivion for decades, they have been reviewed by Peterson (1992a)
2 and translated to English (Peterson, 1992b), as well as summarised and augmented by an F-
3 scale wind damage description adapted to Central Europe by Dotzek et al. (2000). Both
4 Peterson (1992a,b) and Dotzek et al. (2000) show their being well ahead of their time.

5 Yet, not only had the advent of World War II prevented their widespread international
6 application, but also in particular for the USA the sceptical commenting letters by J. B.
7 Kincer, then-Chief of Division of Climate and Crop Weather at the US Weather Bureau, in
8 which he expressed little confidence that ambitious tornado research programs as proposed in
9 Letzmann's guidelines could ever be accomplished. These letters were attached to
10 Koschmieder and Letzmann (1939), and one of them was reproduced and discussed by
11 Peterson (1992b).

12 Based on his streamline analysis, Letzmann (1923, 1925, 1928) had produced images
13 of tree fall pattern along cross-sections of a tornado damage swath for various combinations
14 of the parameters α and G_{max} (cf. Fig. 11 of Peterson, 1992a). To do so, he had assumed that
15 tree fall always occurred in the direction of the instantaneous wind at the location of the tree
16 in the moment of its failure. The same assumption was also made by Holland et al. (2006).
17 Letzmann then categorised the resulting swath patterns into four main types and showed these
18 for six discrete values of the angle of deflection α . This diagram also appeared in the IMO
19 guidelines (Letzmann, 1939) and has been reproduced by Peterson (1992a, Fig. 8), Peterson
20 (1992b, Figs. 1 and 2) and Dotzek et al. (2000, Figs. 1 and 2) and hence is not included here
21 again.

22 When compared to individual swath cross-sections (horizontal rows) of Holland et al.
23 (2006, Figs. 9-15), their resemblance to Letzmann's characteristic swath types is striking. The
24 only significant step forward by Holland et al. (2006) is the inclusion of the detailed tree
25 response model which was unavailable in Letzmann's times. What Letzmann (1923) did
26 investigate, however, was the effect of wind-induced torsion on trees, following the
27 descriptions by Martins (1850) and Wegener (1917). He identified regions inside the vortex
28 which might support twisting off trees by the vortex itself (cf. Fig. 12 of Peterson, 1992a),
29 instead of the more common case where an asymmetric tree crown exposed to a more
30 straight-line wind can also lead to a twisted fracture of the trunk.

31 For completeness, we mention that Letzmann's IMO guidelines also gave an extensive
32 treatment on how to conduct ground and aerial damage surveys to provide the best possible
33 data of the forest damage swath to enable proper reconstruction of the tornado characteristics.
34 Given that the technique of aerial damage surveys was only later taken up and developed to

1 full maturity by Ted Fujita (e. g. Fujita, 1981), we can only speculate what fruitful
2 cooperation could have resulted if Letzmann and Fujita had the chance to work together on
3 tornado damage analysis.

4 5 **3 Conclusions**

6 We welcome the paper by Holland et al. (2006) very much for their addressing a line of
7 research directly linked to Letzmann's investigations in the 1920s and 1930s. However, the
8 following points are important to put Letzmann's work in a proper perspective:

- 9 • Based on the limited information they had on Letzmann's work, Holland et al. (2006)
10 have apparently reinvented parts of Letzmann's analytical tornado vortex model;
- 11 • The full versatility of the analysis by Letzmann (1923, 1925) remains yet to be exploited
12 by Holland et al. (2006) and other groups addressing tornado damage assessments;
- 13 • We have provided here the necessary background and references to Letzmann's work and
14 thus hope to stimulate further use of Letzmann's results for development or refinement of
15 forest damage models such as that of Holland et al. (2006).

16 We are confident that Letzmann's achievements still can foster contemporary tornado
17 research. The well-documented forest damage swath of the 2 October 2006 F3 Quirla tornado
18 in Germany might serve as a test case to apply the Holland et al. (2006) model over hilly
19 terrain.

20 21 **Acknowledgements**

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23 available the Letzmann legacy and to Johannes Letzmann's grand-nephew Michael for
24 providing additional information and material on Letzmann's life. Peer Hechler of the
25 German Weather Service (DWD) and member of WMO's Climatological Commission
26 performed investigations at WMO to trace the status of IMO resolutions.

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Figure captions

Fig. 1: (a) Söderberg's apparatus for graphical solution of differential equations, as used by Sandström (1909, Fig. 3) for construction of isogones and streamlines, and (b) example of graphical solution of the streamline equation $dy/dx = \tan [3\pi \sin (x^2 + y^2)^{1/2}]$ from Sandström (1909, Plate 32).

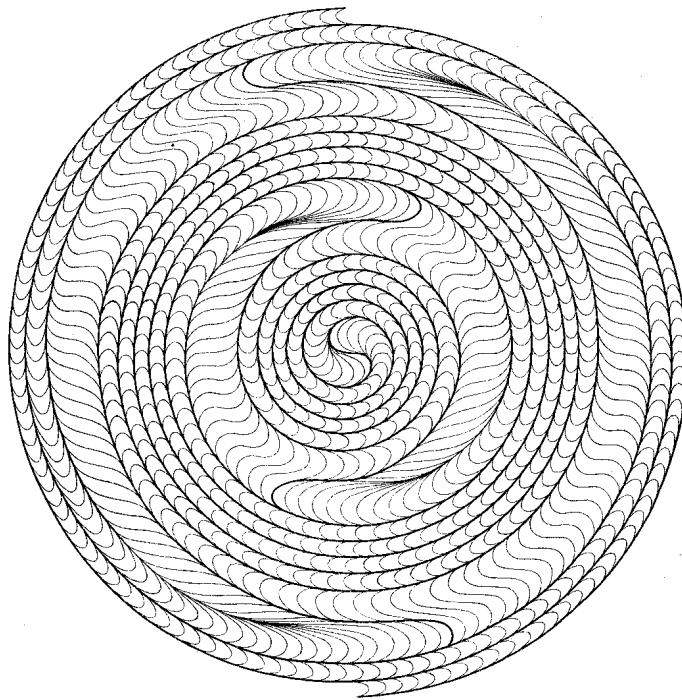
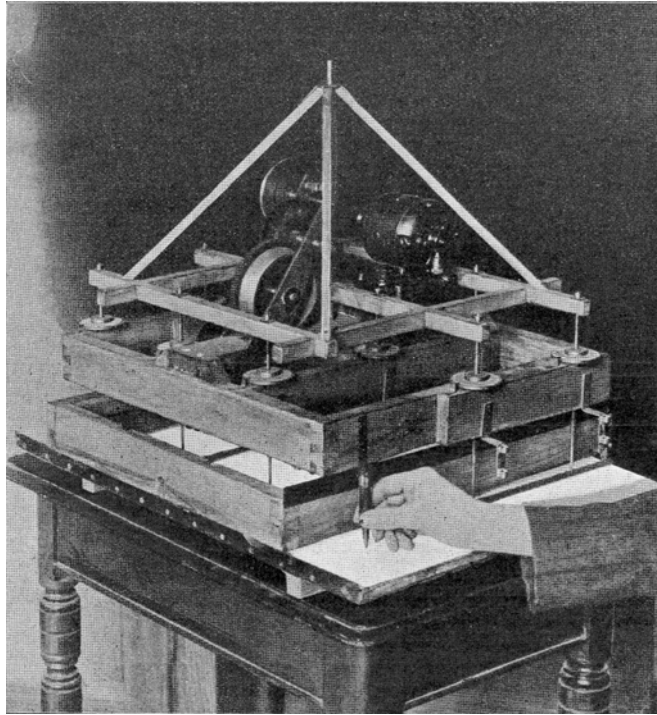


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