Power Laws in the Atmosphere

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Motivation: Atmospheric complex phenomena

Synoptic scale
Source: NOAA

Meso-scale
Source: NASA

Convective scale
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Former Work

Topic:
Investigation of frequency distributions of low pressure systems on different scales

Central question:
How to find an appropriate intensity parameter?

First idea:
Parameter definition based on the horizontal equation of motion
Introduction of an energy of displacement (Schielicke and Névir, 2009)

How to describe vortices with help of the horizontal equation of motion?

Definition of a mass-specific energy of displacement $E$:

Radial path integral at the moment of maximum intensity describes the mass-specific work that was necessary for the generation of the system.

$$E := \int_{0}^{R} a(r) \, dr = -\int_{P_0}^{P} \rho^{-1} \, dp$$

with $a(r) := (v(r))^2 / r + fv(r)$

Explicit expression depends on the scale

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Low Pressure System
- Tornadoes
- Tropical Cyclones
- Extratropical Cyclones

Prevailing Balance
- Cyclostrophic
- Gradient wind
- geostrophic

Energy of Displacement
- $E=v^2/2$
- $E=v^2+fvR$
- $E=fvR$
Frequency distributions of vortices concerning their energy of displacement expression (Schielicke and Névir, 2009)

**Questions:**
- Why do we observe similar exponential behavior?
- What is the meaning of the decay rate?

Fig. 4. Summary of density-intensity (energy of displacement) distributions per year of tornado data (1950–1999) and cyclone data (1958–1997), containing the summation of hurricanes and extra-tropical depressions (see Table 3). The distributions show the same exponential behavior, represented by the same characteristic, universal energy of displacement scale of \( E_t \approx 1000 \text{ m}^2/\text{s}^2 \) over the whole range.

Former and current Work

Central question:

Why can’t we find power laws for atmospheric features although other criteria of complex systems match for the atmosphere?

Established example: Gutenberg-Richter law of earthquakes

Is it possible to apply the seismic moment concept to the atmosphere?
Introduction of an atmospheric moment
(Schielicke and Névir, 2011)

Seismic moment concept combines intensity (stress drop) with geometric properties.

Application to atmospheric vortices:

Combination of the local intensity (the energy of displacement) with the mass affected during the life-cycle:

\[ M_a := \bar{\rho}V E = -\bar{\rho}V \int_{P_0}^{P} \rho^{-1} dp \]

Interpretation:
- Proportional to the total, mass-related work of generation.
- Proportional to a heat amount due to external forcing?
- Consideration of whole life-cycle; process-related

Application of moment concept to tornadoes

- Atmospheric moment $M_a$ as product of total during life time affected mass $M$ and energy of displacement $E$:

$$M = \bar{\rho}V = \bar{\rho}AL / C^{hv}$$

$$E = \frac{v_F^2}{2}$$

- Explicit expression for tornadoes:

$$M_a \sim AL\bar{\rho}\left\langle v_F^2 \right\rangle / 2$$

A: Area at max. intensity
L: Path length
$\rho$: density
v: velocity
$C^{hv}$: constant relating hor. and vertical extent
Frequency distributions of tornadoes and earthquakes concerning their moments (Schielicke and Névir, 2011)

Finally power laws!

- How do other vortices and their properties behave?
- What is the meaning of the exponents?

Exponents for Tornadoes: 1.2
Earthquakes: 1.7

Application of atmospheric moment concept to synoptic-scale cyclones

- Local intensity (energy of displacement) can be estimated by $E \approx \rho^{-1} \Delta P$
- $\Delta P$: Pressure deficit between vortex center and environment
  
  $M_a := \bar{\rho} V E = -\bar{\rho} V \int_{P_0}^{P} \rho^{-1} dp$

- Estimation of total volume $V = L \times 2R \times H \approx LR^2 = LA$
- $\bar{\rho} \approx \text{const.}$

Explicit expression of atmospheric moment for synoptic-scale vortices:

$M_{a,cyc} \sim LA \Delta P$

- $A$: Area at maximum intensity
- $L$: Path length
- $\Delta P$: Pressure deficit at maximum intensity
  
  $\rho^{-1} \Delta P = -g_0 \Delta h$, $g_0 = 9.80665 \text{m/s}^2$
ERA-Interim data

- Spatial resolution of about 1.5°
- Temporal resolution: $\Delta t=6h$
- Time period: Jan-Dec 1989,
- 1000 hPa geopotential height field,
- 80°N-80°S
Method of cyclone tracking and area estimation

- **Identification** of cyclones as local minima in the geopotential height field

- **Tracking**: Nearest-neighbor search method (similar to Blender et al., 1997)

- **Area estimation**: outermost closed isobar (with help of contourplot algorithm of gnuplot)

Only a first guess!
Probability density distributions of lows – first results

Results:
- Covers a large range of moments.
- Power law behavior?

Fig. 5. Comparison of probability density distributions of tornadoes (1950–2006) and earthquakes (1976–2005) concerning their moments in a log–log plot. Linear fits have been applied to the data. The fitting ranges are indicated by the filled symbols.
Probability density distributions of lows – first results

\[ y = -0.6932x - 6.5773 \]

\[ R^2 = 0.999 \]

\( \approx 50\% \)

41133 Tracks in 1989, ERA-Interim data
Probability density distributions of lows – first results

41133 Tracks in 1989, ERA-Interim data

Cyclones (global, 1989)

$y = -0.6932x - 6.5773$

$R^2 = 0.999$

$\approx 50\%$

$y = -1.3256x + 5.6534$

$R^2 = 0.9953$

$\approx 50\%$
Probability density distributions of cyclones – first results

Number of Tracks in 1989 N/S-Hemisphere;
$\Delta t \geq 2$  North: 11845; South: 29291 Tracks,
$\Delta t \geq 4$  North: 5252; South: 13376 Tracks

$y = -1,3256x + 5,6534$
$R^2 = 0,9953$
Area

$N_{\text{total}}$ (South) = 60247  
$N_{\text{total}}$ (North) = 23251

6th ECSS, Palma (Mallorca),  
Path length

\[ \log(\text{dn/dL}) \text{ with n=N_i/N_{total}} \]

\[ \log(\text{Path length L [km]}) \]

- \( \Delta t \geq 2 \)
- \( \Delta t \geq 4 \)

North
South
North
South

6th ECSS, Palma (Mallorca),
Local maximum intensity

Northern Hemisphere

Southern Hemisphere

Energy of displacement

$N / N_{\text{total}}$ with $N_{\text{local}}=11845$

Energy of displacement

$N / N_{\text{total}}$ with $N_{\text{local}}=29289$

- North, $\Delta t >= 2$
- North, $\Delta t >= 4$
- North all

- South, $\Delta t >= 2$
- South, $\Delta t >= 4$
- South all
Comparison N/S-hemisphere intensity

$y = 7753.2e^{-0.0008x}$

$R^2 = 0.9853$

$y = 5275.9e^{-0.0011x}$

$R^2 = 0.9842$
Results

Exponents for:
- Tornadoes: 1.2
- Earthquakes: 1.7
- Cyclones: 1.3

Fig. 5. Comparison of probability density distributions of tornadoes (1950-2006) and earthquakes (1976-2005) concerning their moments in a log-log plot. Linear fits have been applied to the data. The fitting ranges are indicated by the filled symbols.