Towards a Power Law Distribution of Tornadoes and Cyclones and the relation to the Gutenberg-Richter Law of Earthquakes

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I. INTRODUCTION

Various definitions of the *intensity* of atmospheric vortices exist. These definitions are based on different (physical) parameters, e.g. the intensity of tornadoes is defined by their damage-related wind speeds (Fujita-scale, Enhanced-Fujita-scale), cyclones are mostly investigated concerning their minimum central pressure, pressure gradient or concerning the maximum occurring wind speed.
Do these local parameters really describe the systems as a whole? What methods are applied in other geosciences to handle this problem?
Peering into seismology, one finds the concept of the seismic moment, which contains all information about the physical parameters of an earthquake source. Seismic moment as a measure of the magnitude or strength of an earthquake is also related to the energy released during the quake (e.g. Stein and Wysession, 2002). Analogous to this concept a moment of tornadoes and other atmospheric vortices is introduced. An analysis of the frequency distribution of tornadoes will be presented and compared to the frequency distributions of earthquakes (known as Gutenberg-Richter-law).

II. THEORY

The seismic moment tensor contains nine force couples representing the equivalent body forces for seismic sources of different geometries. Because of the symmetry of the seismic moment tensor there are only six independent elements. The scalar seismic moment M_0 equates the magnitude of the seismic moment tensor and is given by

$$M_0 = (L/C)S\Delta\sigma \quad , \tag{1}$$

where L is the dimension of the fault, C is a nondimensional shape factor, S is the fault area and $\Delta\sigma$ is the stress drop (Kanamori and Anderson, 1975). The scalar seismic moment is proportional to the energy released.
A similar expression of the moment of atmospheric vortices is proposed in which the connecting link between seismic and atmospheric moment is given by the stress tensor in the Navier-Stokes-equation. The atmospheric moment is estimated as

$$M_a = LA\Delta p \quad , \tag{2}$$

where L is the track length, $A = \pi R^2$ is the area of the vortex with radius R, and Δp is the pressure deficit of central and environmental pressure. Note that LR gives the total affected area of the vortex during its lifetime.
The advantage of the moment concept is that it not only considers the local intensity but also takes the affected volume of the systems into account and therefore describes the vortex as a globally integrated structure. This changes the point of view of analysis from a consideration of the local state to a consideration of the process involved.

III. DATA AND RESULTS

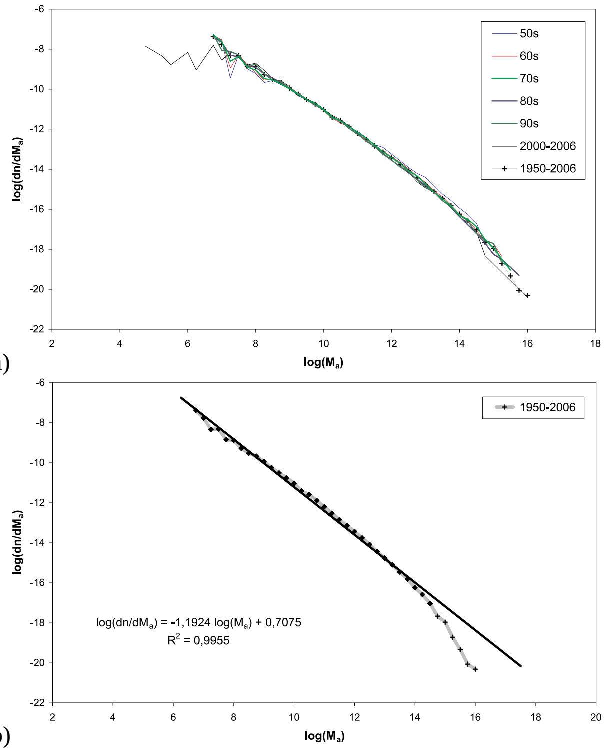
Data of the US-tornado database in the period of 1950 to 2006 (SPC, 2009) has been used to analyse probability density functions (pdfs) of tornadoes concerning their atmospheric moment. Pressure deficit Δp in tornadoes was calculated using its relation to the kinetic energy due to the cyclostrophic balance. Figure 1(a) shows the log-log-plot of the pdfs for different decades. The similarity between the decades is evident just like the nearly overall linear behaviour in the log-log-plot and therefore power-law behaviour of the pdfs over at least 5 orders:

$$\frac{dn}{dM_a} = \alpha M_a^{-\beta} \quad , \tag{3}$$

where $n_i = N_i/N_{total}$ gives the percentage of tornado cases N_i of class i relative to the total number of tornadoes. α is a constant. The exponent β of the power law is approximately $\beta \approx 1.2$ (see figure 1(b)) and is therefore somewhat smaller than the exponent of the power-law distributed pdfs of earthquakes (≈ 1.67) concerning their seismic moment (not shown in this abstract).

IV. CONCLUSIONS

A moment of atmospheric vortices analogous to the seismic moment of earthquakes is proposed. Probability density functions of tornadoes concerning their moment show power law behaviour. Compared with earthquakes the exponent is slightly smaller but of comparable order.
In a previous work of the authors, successful attempts have been made to find a physical parameter that unifies frequency distributions of different atmospheric low pressure systems (Schielicke and Névir, 2009). On basis of the horizontal, frictionless equation of motion an energy of displacement has been defined, which represents the (mass-specific) work that is necessary to generate the low pressure system.



(a)

(b)

FIG. 1: Log-log-plot of the pdfs of tornadoes: (a) pdfs of the decades from 1950-1999 and the years from 2000-2006, (b) pdf of the total period with linear regression (regression range is indicated by closed rectangles).

Analysing frequency distributions of tornadoes and cyclones, respectively, concerning their energy of displacement showed similar exponential behavior with the same (universal) characteristic parameter with a value of approximately $E_u \approx 1000 J/kg$. Additionally, the mass-specific energy of displacement is related to the mass-specific pressure deficit Δp used in this work.
The presented work gives an expansion from a local, mass-specific view to a global, integrated, mass-related parameter (moment) that regards the system as a whole considering life-time and integrated affected mass. The analysis of other low pressure systems concerning their atmospheric moment is planned in the future.

V. ACKNOWLEDGMENTS

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VI. REFERENCES

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