On the Theory of Statistical Intensity Distributions of Tornadoes and other Low Pressure Systems

Lisa Schielicke[1], Peter Névir[1]

[1]*Freie Universität Berlin, Institut für Meteorologie*
Carl-Heinrich-Becker-Weg 6-10, D-12165 Berlin, Germany
lisa.schielicke@met.fu-berlin.de
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I. INTRODUCTION

Tornado intensity distributions of various countries have been described by Weibull-distributions (Dotzek et al., 2003). These statistics count the number of tornadoes in classes of the Fujita intensity scale. Our suggestion is to use the mass specific Fujita-energy ($1/2v_F^2$, where v_F is the Fujita velocity) instead of the Fujita scale. Energy is a common physical quantity, which allows a comparison of the tornado distributions with distributions of other phenomena, like Hurricanes and extra-tropical lows. The result is an exponential distribution for the number of tornadoes in classes of energy (N(E)). Theoretical aspects can then be used to obtain a relation between the number of phenomena with respect to their central surface pressure.

II. THEORY

The radial component of the equation of motion in natural coordinates describes the balance of mass specific forces. The pressure gradient force is balanced by centrifugal force, Coriolis force, and friction. Integrating over the radius of a general circularly shaped pressure system under assumption of constant temperature T leads to a generalized Boltzmann-distribution.

$$P(E) = P_0 e^{-\int F(r)dr/RT} \qquad (1)$$

Provided that the distributions of the different pressure systems like extra-tropical lows, hurricanes, and tornadoes show an exponential behaviour the empirical number-intensity relation is given by

$$N(E) = N_0 e^{-\beta \int F(r)dr} \quad . \qquad (2)$$

Combining theoretical (1) and empirical (2) aspects, a power law of atmospheric pressure anomalies of all scales is the result. A power law could be a sign for self-organized behaviour (Bak, 1997).

$$N(P) = N_0 \left(\frac{P}{P_0}\right)^{RT/E_u} \qquad (3)$$

$E_u=1/\beta$ is the characteristic energy scale that describes the relation between number and intensity of the different pressure systems.

The solutions of the integrals over the forces in the exponents of equations (1) and (2), neglecting friction, are shown in table I.

	Solution of $\int F(r)dr$	Balances
Tornadoes	$v^2/2$	cyclostrophic
Hurricanes	$v^2 + fvr$	gradient wind
Extratrop. lows	$fv_g r$	geostrophic

TABLE I: Solutions of the force integrals and the prevailing balance of forces

III. RESULTS AND CONCLUSIONS

The number-intensity distribution N(E) of tornadoes is shown and compared to Weibull statistics. It should be noted, that the exponential behaviour fits best for strong and violent tornadoes (F2-F5). A first estimate reveals that the hypothetical cases of F-2 and F-1 sub-critical tornadic circulations are not as clearly noticeable as in the Weibull fit. Also the number of potential F6 tornadoes seems to be lower as in the Weibull fit.

The comparison of the central pressure in a tornado with the environmental pressure can be used to discuss how many sub-critical tornado vortices can form under these circumstances.

The generalization of this theory on extra-tropical lows and hurricanes is interesting because it could help understanding some aspects of emergence or concurrence of the atmospheric pressure systems. Generally, hurricanes and extra-tropical cyclones exhibit the same characteristic energy scale of about $E_u = 1/\beta \approx 1.1\cdot10^3 m^2/s^2$, as do tornadoes. The physical meaning of this result is not entirely understood so far and will be a matter of future work.

The characteristic mass specific energy E_u of the three phenomena can be used to calculate characteristic velocities. The characteristic velocities differ because of the different prevailing balances of forces (see table II).

	Velocity in relation to characteristic energy	Characteristic velocity
Tornadoes	$v = \sqrt{2E_u}$	$v = 47ms^{-1}$
Hurricanes	$v = -\frac{fr}{2} + \sqrt{\frac{(fr)^2}{4} + E_u}$	$v = 23ms^{-1}$
Extratrop. lows	$v = \frac{E_u}{fr}$	$v = 11ms^{-1}$

TABLE II: Characteristic velocities, $E_u = 1.1\cdot10^3 m^2/s^2$, different radii for hurricanes (r=500km) and cyclones (r=1000km)

V. REFERENCES

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