DERIVATION OF PHYSICALLY MOTIVATED WIND SPEED SCALES

Nikolai Dotzek[1,2]

[1]*DLR-IPA, Oberpfaffenhofen, Germany, e-mail address nikolai.dotzek@dlr.de*
[2]*ESSL, Wessling, Germany, e-mail address nikolai.dotzek@essl.org*

(Dated: May 1, 2007)

I. INTRODUCTION

A class of new wind speed scales is proposed which rely on physically relevant quantities like mass flux density, energy density (pressure), or energy flux density. These so-called Energy- or E-scales can be applied to wind speeds of any intensity. Details are provided by Dotzek (2007).

Development of wind speed scales has long been a subject of research, and Fujita (1981) has provided a review of the field focusing on those scales which were designed to describe the most intense wind phenomena on earth: Tornadoes, downbursts, and tropical cyclones. Inherently, the task of devising scales for high wind events can be tackled from two sides:

(i) wind speed-based, and
(ii) damage-based.

The former approach is usually taken by the atmospheric sciences, while the latter reflects more the standpoint of wind engineering. However, the conceptual difference and partial incompatibility of both approaches has led to considerable controversy and confusion, primarily because even wind speed-based scales must usually rely on post-event damage surveys, due to the scarcity of in situ wind measurements, at least in tornadoes and downbursts.

The difference between approaches (i) and (ii) above can be substantial, as wind speed-based scales are in general concerned about the maximum winds that can physically occur for a given wind phenomenon, and in particular about what its maximum (local) intensity (wind speed) was.

Damage-based scales, however, focus on determining the minimum wind speed necessary to cause the observed damage to individual man-made structures or vegetation. Also, a likely upper bound of wind speeds can be estimated in those cases in which undamaged structures remain, for which apparently their critical damaging wind speed level had not been attained in the storm.

Three wind speed scales are frequently used in meteorology, wind engineering and related sciences: the Beaufort (B), Fujita (F), and TORRO (T) scales. The relationship between velocity v and the scale value X in these various scales can be described as

$$v(\mathrm{X}) = v_* \, (\mathrm{X} - \mathrm{X}_0)^{3/2} \quad , \qquad (1)$$

which may also be used for an approximation of the Saffir-Simpson (S) scale mainly applied to hurricane winds over the Atlantic basin. For the Fujita scale, Eq. (1) becomes

$$v(\mathrm{F}) = 6.302 \text{ m s}^{-1} \, (\mathrm{F}+2)^{3/2} \quad . \qquad (2)$$

The F, T, and S-scales classify the physically possible velocity range for tornadoes, downbursts, and tropical cyclones. This makes them applicable worldwide in a consistent way – an important point in climatological analysis. Yet, the question if the exponent 3/2 in Eq. (1) is the best possible choice was often raised.

II. DERIVATION OF THE E-SCALE

One flaw of, for instance, the current F- and T-scales is that they distinguish more than one sub-critical class (so, $\mathrm{X}_0 < -1$). Therefore, the first step is to require $\mathrm{X}_0 = -1$ as default for any new high wind speed scale in order to avoid unwanted detail with sub-critical winds (scales considering any wind speed relevant, like the B-scale, have $\mathrm{X}_0 = 0$).

To meet also additional requirements for the new scales, any formulation should be based on physical observables, like the maximum horizontal wind speed v (or momentum density), maximum values of kinetic energy ($\propto v^2$) or energy-flux density ($\propto v^3$). They bear more physical relevance than a formal scale variable X and, depending on structural characteristics, v^2 or v^3 are directly related to wind load and damage (cf. Dotzek et al., 2005; Dotzek, 2007):

$$M = \rho \, v \quad , \quad [\mathrm{M}] = \mathrm{kg\ m^{-2}\ s^{-1}} \quad , \text{ mass flux density,} \qquad (3a)$$
$$E = \rho / 2 \, v^2 \quad , \quad [\mathrm{E}] = \mathrm{J\ m^{-3}} = \mathrm{Pa} \quad , \text{ energy density,} \qquad (3b)$$
$$P = \rho / 2 \, v^3 \quad , \quad [\mathrm{P}] = \mathrm{W\ m^{-2}} \quad , \text{ energy flux density.} \qquad (3c)$$

Accordingly, we propose the following generic type of scaling, henceforth termed the "Energy-scale" or E-scale:

$$X_* \, (\mathrm{X} - \mathrm{X}_0)^n = a_x \, v^n \quad => \quad v(\mathrm{X}) = v_* \, (\mathrm{X} - \mathrm{X}_0) \quad ,$$
$$v_* = [a_x^{-1} \, X_*]^{1/n} \quad . \qquad (4)$$

Application of this E-scaling and using $\mathrm{X}_0 = -1$ leads to:

$$M_* \, (\mathrm{X} + 1) = \rho \, v \quad => \quad v(\mathrm{X}) = v_* \, (\mathrm{X} + 1) \quad ,$$
$$v_* = \rho^{-1} M_* \quad , \qquad (5a)$$

$$E_* \, (\mathrm{X} + 1)^2 = \rho / 2 \, v^2 \quad => \quad v(\mathrm{X}) = v_* \, (\mathrm{X} + 1) \quad ,$$
$$v_* = [2 \, \rho^{-1} E_*]^{1/2} \quad , \qquad (5b)$$

$$P_* \, (\mathrm{X} + 1)^3 = \rho / 2 \, v^3 \quad => \quad v(\mathrm{X}) = v_* \, (\mathrm{X} + 1) \quad ,$$
$$v_* = [2 \, \rho^{-1} P_*]^{1/3} \quad . \qquad (5c)$$

For this E-scaling, characterised by linear $v(\mathrm{X})$ functions, all values v_* in Eqs. (5a-c) are constants. This means that for externally specified critical values of M_*, E_*, or P_*, the scaling velocity v_* can be computed (calibration). Or, for any specification of v_* (like with the present F-, S- or T-scales), the corresponding physical quantities M_*, E_*, or P_* can be evaluated for comparison:

$$v_* = \rho^{-1} M_* = [2\rho^{-1} E_*]^{1/2} = [2\rho^{-1} P_*]^{1/3} = v'_* \quad . \qquad (6)$$

Note that the Mach- or M-scale for wind speeds from 0 to the supersonic range is a special case of an E-scale with externally specified v_* and is also named $\mathrm{E_M}$-scale here:

$$v(\mathrm{M}) = v_* \; \mathrm{M} \quad ,$$
$$v_* = [\kappa R \, T]^{1/2} = [\kappa \rho^{-1} p]^{1/2} \approx 340 \text{ m s}^{-1} \quad . \quad \mathrm{E_M}\text{-scale} \qquad (7)$$

Herein, M denotes the Mach number, v_* is the speed of sound, and $\kappa = c_p/c_v$, $R = c_p - c_v$, T, and p have their usual thermodynamic meanings. Evaluation and calibration of the E-scales is further described in detail by Dotzek (2007).

To gain acceptance for the new E-scale, existing data based on, for example, F- or T-scale ratings should be readily convertible to the E-scale and also keep the workload for re-rating recorded events manageable.

Any existing scale obeying Eq. (1) can be converted into the E-scale of Eq. (4) and vice versa by these transformations between $v(\mathrm{E}) = v_*\ (\mathrm{E} - \mathrm{E}_0)$ and the $v(\mathrm{X})$ relation:

$$\mathrm{E}' = X_* \,/\, v_*\ (\mathrm{X} - \mathrm{X}_0)^{3/2} + \mathrm{E}_0 \quad , \tag{8a}$$

$$\mathrm{X}' = [v_* \,/\, X_*\ (\mathrm{E} - \mathrm{E}_0)]^{2/3} + \mathrm{X}_0 \quad , \tag{8b}$$

wherein E and E_0 denote the E-scale variable and offset. We illustrate the conversion procedure from F- to E_F-scale here (cf. Table I). E0 = -1 in this case, and the choice of initial v_* values was made for compatibility of the main E_F-scale thresholds to those of the F-scale, to facilitate conversion of ratings based on F-scale to the E-scale definitions.

The E_F-scale comprises the same number of classes as the F-scale, yet contains only one sub-critical class and hence one more class in the relevant range F0 to F5. The enhanced resolution mainly sets in above the F4 threshold, i. e. [F4, F5] => [E4, E5, E6]. This is also the intensity range for which the F-scale forum (McDonald, 2002, cf. www.april31974.com/fujita_scale_forum.htm) claimed the largest demand for changes in the choice of scale class boundaries.

As a conclusion of Table I, conversion of the US tornado intensity data from F- to E_F-scale would mainly require review of the recorded F5 events, which only amounted to roughly 10 per decade in the 20th century (cf. Dotzek et al., 2003). In the same period, about 80 F4 tornadoes per decade were recorded in the USA, of which only the stronger ones would have to be re-rated to E_F-scale based on the available case information. So even for the world's largest tornado database, the workload involved to change to E-scale would indeed remain manageable.

F	v(F) in m s^{-1}	E_F'	E_F integer
-2	0.0	-1.00	-1
-1	6.3	-0.65	-1
0	17.8	0.00	0
1	32.7	0.84	1
2	50.4	1.83	2
3	70.5	2.95	3
4	92.6	4.20	4, 5
5	116.7	5.55	5, 6
6	142.6	7.00	7

TABLE I: Conversion of F- to E_F-scale thresholds using $v_{*,\mathrm{E}} = 17.825$ m s^{-1} and $v_{*,\mathrm{F}} = 6.302$ m s^{-1} according to Eq. (8) Note that only the F4, F5 classes have to be sub-divided into E4, E5, E6 classes in converting F-scale to E_F-scale data.

Relying on physical quantities was also one motivation for Emanuel (2005) to develop the Power Dissipation Index (PDI) for tropical cyclones. It is evident that an E-scale based on the scaling quantity P_* is directly linked to the integral measure PDI. Also, to advance from scales based on observed wind damage to the E-scale would be a similar step forward as switching from the Mercalli to the Gutenberg-Richter earthquake scale in geophysics. Mercalli's scale was based on eyewitness and damage reports, with shortcomings very similar to those encountered in present wind event ratings. The Gutenberg-Richter scale, however, is an energy scale. Adopting the E-scale and applying it to the PDI concept could provide a way to measure the total energy expended in a wind event, and this would be much more meaningful than any present point measurement or damage assessment.

The respective merits of the E-scale framework to the recently proposed "Enhanced Fujita-scale" (EF-scale, cf. Doswell et al., 2007) are also discussed by Dotzek (2007).

III. CONCLUSIONS

Our analysis led to a new type of wind speed scale, named Energy-scale or E-scale. Especially the E_F-scale is proposed to serve as a physics-based alternative to the F-scale. Yet, any scale obeying Eq. (4) is an E-scale and bears the following useful properties:

- The E-scale is based on physical quantities and hence can be calibrated;
- E-scale versus wind speed relations are always linear;
- The E_F-scale comprises the same number of classes as the F-scale, yet one more class in the relevant range F0 to F5. The enhanced resolution mainly sets in above the F4 threshold, i. e. [F4, F5] => [E4, E5, E6], so F-scale data is easy to convert to E_F-scale;
- The F-scale thresholds F-2, F0, and F6 are exactly mapped to E-1, E0, and E7, while the F3 and E3 thresholds are nearly identical;
- The E-scale concept can also help to overcome the present zoo of different scales for winds from storm to hurricane intensity.

The next step is calibration of the E-scales of which only the Mach-scale E_M is presently calibrated. Should this be done, conversion among recalibrated E-scales would be easy due to their linear wind speed versus scale relationship.

IV. AKNOWLEDGMENTS

The work on the E-scale concept started in the Christmas break 2004/2005, kindly supported by the patience of the author's family. The author is further grateful to Chuck Doswell for stimulating discussions on this matter and to Jeff Trapp and Kerry Emanuel for helpful comments.

V. REFERENCES

Doswell, C. A., H. E. Brooks, and N. Dotzek, 2007: On the implementation of the Enhanced Fujita scale in the USA. *This volume*.

Dotzek N., 2007: Derivation of physically motivated wind speed scales. *J. Atmos. Sci*, submitted.

Dotzek, N., J. Grieser, and H. E. Brooks, 2003: Statistical modeling of tornado intensity distributions. *Atmos. Res.*, 67-68, 163-187.

Dotzek, N., M. V. Kurgansky, J. Grieser, B. Feuerstein, and P. Névir, 2005: Observational evidence for exponential tornado intensity distributions over specific kinetic energy. *Geophys. Res. Lett.*, 32, L24813, doi:10.1029/2005GL024583.

Emanuel, K., 2005: Increasing destructiveness of tropical cyclones over the last 30 years. *Nature*, 436, 686-688. doi:10.1038/nature03906.

Fujita, T. T., 1981: Tornadoes and downbursts in the context of generalized planetary scales. *J. Atmos. Sci.*, 38, 1511-1534.

McDonald, J. R., 2002: Development of an enhanced Fujita scale for estimating tornado intensity. Preprints, *21st Conf. on Severe Local Storms*, Austin. Amer. Meteor. Soc., Boston, 174–177. [ams.confex.com/ams/SLS_WAF_NWP/21SLS/abstracts/¬47974.htm]